

# ON EXTRACTION OF THE TOTAL PHOTOABSORPTION CROSS SECTION ON THE NEUTRON FROM DATA ON THE DEUTERON

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An improved procedure is suggested for finding the total photoabsorption cross section on the neutron from data on the deuteron at energies  $\lesssim 1.5$  GeV. It includes unfolding of smearing effects caused by Fermi motion of nucleons in the deuteron and also takes into account non-additive contributions to the deuteron cross section due to final-state interactions of particles in single and double pion photoproduction. This procedure is applied to analysis of existing data.

## Introduction

This work was motivated by recent preliminary results from the GRAAL experiment on the total photoabsorption cross section off protons and deuterons at photon energies  $\omega = 700\text{--}1500$  MeV [1–4] and their implications for the neutron. An intriguing feature of the new data is that they indicate an approximately equal and big strength of photoexcitation of the nucleon  $F_{15}(1680)$  resonance off both the proton and neutron (as seen, in particular, in Fig. 5 in Ref. [4]). Meanwhile this strength was found small for the neutron in many previous studies (see, e.g., [5, 6]). Particle Data Group [7] quotes the following branching ratios of  $N^* = F_{15}(1680)$  to  $\gamma N$ :

$$\begin{aligned}\text{Br}(N^* \rightarrow \gamma p) &= 0.21\text{--}0.32\%, \\ \text{Br}(N^* \rightarrow \gamma n) &= 0.021\text{--}0.046\%.\end{aligned}\tag{1}$$

Irrespectively on whether the old or new data are correct, it seems timely to (re)consider procedure commonly used to find cross sections off the neutron from the deuteron data.

This procedure was described in detail by the Daresbury group [6] who performed measurements of the total photoabsorption cross sections  $\sigma_p$  [5] and  $\sigma_d$  [6] at energies between 0.265 and 4.215 GeV. In the nucleon resonance energy region they made an Ansatz that

$$\sigma_d(\omega) = F(\omega)[\sigma_p(\omega) + \sigma_n(\omega)].\tag{2}$$

Here the factor of  $F(\omega)$  was introduced in order to take into account smearing effects due to Fermi motion of nucleons in the deuteron. This factor was found by numerical integration of the proton cross sections using known momentum distribution of nucleons in the deuteron and then equally applied to the neutron. Finally, the neutron cross section was found, point by point, with the step of 25 MeV, from the corresponding deuteron cross section at the same energy using Eq. (2).

An evident drawback of the Ansatz (2) is that smearing effects are assumed to be the same for the proton and neutron, what cannot be true in case the energy dependencies of  $\sigma_p(\omega)$  and  $\sigma_n(\omega)$  are different.

The second problem is that smearing of the cross section makes it impossible to relate individual nucleon cross sections  $\sigma_N(\omega)$  with  $\sigma_d(\omega)$  at the same energy and thus to apply the point-by-point procedure. Instead, some average of  $\sigma_N(\omega)$  over a finite energy interval can only be found. In other words, a justified unsmearing procedure should be applied there.

The third point is that non-additive corrections related mostly with final state interactions have been neglected in Eq. (2). Brodsky and Pumplin [8] estimated these corrections at high energies ( $\omega \gtrsim 2$  GeV) assuming that high-energy photoproduction on the nucleon is dominated by diffractive photoproduction of vector mesons ( $\rho$ ,  $\omega$ ,  $\phi$ ) which then interact with the second nucleon.

Such corrections have been included in the analysis of high-energy part of the Daresbury data [6] (as well as in studies of photoabsorption off protons and deuterons at energies 20–40 GeV [9]). At lower energies, including energies of GRAAL, the corrections related with vector meson production are small. Nevertheless, other photoproduction channels still might be important. This is indeed the case as explained below. To our knowledge, no estimates of the non-additive corrections to Eq. (2) have been yet done at energies of the GRAAL experiment.

In this work we improve the procedure of [6] in all the above three lines.

## Fermi smearing (folding)

We begin with rewriting Eq. (2) more accurately as

$$\sigma_d(\omega) = \hat{F}[\sigma_p(\omega) + \sigma_n(\omega)] + \Delta\sigma_{pn}(\omega). \quad (3)$$

Here  $\hat{F}$  is a linear integral operator that smears individual nucleon cross sections in accordance with Fermi motion of nucleons in the deuteron;  $\Delta\sigma_{pn}$  is a non-additive correction to be discussed later. The first two terms in Eq. (3) arise from diagrams of impulse approximation (like those in Fig. 1) when interference effects are omitted. We neglect here off-shell effects for intermediate nucleons  $\tilde{N}$  because the binding energy of nucleons in the deuteron is rather small (2.2 MeV).

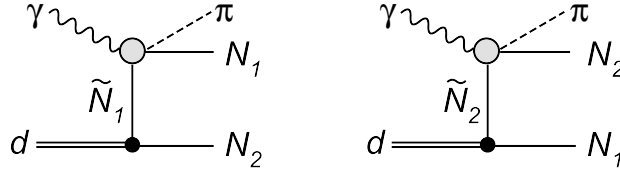


Figure 1: Diagrams of impulse approximation for  $\gamma d \rightarrow \pi NN$ . Antisymmetrization over  $N_1$  and  $N_2$  is not shown.

A simple analysis of diagrams of impulse approximation shows [10] that the smearing operator, in nonrelativistic approximation over nucleons in the deuteron, is reduced to

$$\hat{F}\sigma_N(\omega) = \int W(p_z) \frac{\omega^{\text{eff}}}{\omega} \sigma_N(\omega^{\text{eff}}) dp_z. \quad (4)$$

Here

$$\omega^{\text{eff}} = \omega \left(1 - \frac{p_z}{M}\right) \quad (5)$$

is the effective (Doppler shifted) energy for the moving intermediate nucleon  $\tilde{N}$  of the mass  $M$  provided its longitudinal (along the photon beam) momentum is equal to  $p_z$ .  $W(p_z)$  is the longitudinal momentum distribution of nucleons in the deuteron,

$$W(p_z) = \int |\psi(p)|^2 \frac{d^2 p_\perp}{(2\pi)^3}, \quad (6)$$

and the factor  $\omega^{\text{eff}}/\omega$  takes into account a change in the photon flux seen by the moving nucleon. As in Ref. [6], we use in the following a simplified deuteron wave function (Hulthén [11]),

$$\psi(r) = \frac{k}{r} (e^{-ar} - e^{-br}), \quad \int_0^\infty |\psi(r)|^2 4\pi r^2 dr = 1, \quad (7)$$

with  $a = 45.7 \text{ MeV}/c$ ,  $b = 260 \text{ MeV}/c$  and  $k^2 = ab(a+b)/[2\pi(a-b)^2] = 12.588 \text{ MeV}/c$ . In the  $p$ -space

$$\psi(p) = 4\pi k \left( \frac{1}{a^2 + p^2} - \frac{1}{b^2 + p^2} \right), \quad (8)$$

so that the function  $W(p_z)$  is

$$W(p_z) = 2k^2 \left( \frac{1}{A} + \frac{1}{B} - \frac{2\ln(B/A)}{B-A} \right), \quad \int W(p_z) dp_z = 1, \quad (9)$$

where  $A = a^2 + p_z^2$  and  $B = b^2 + p_z^2$ . This function is shown in Fig. 2 together with a distribution obtained with a realistic (CD-Bonn) wave function [12]. In actual calculations we cut off momenta  $|p_z| > p_{\text{cut}} = 200 \text{ MeV}/c$  where  $W(p_z)$  becomes quite small and the momentum  $p_z$  remains nonrelativistic.

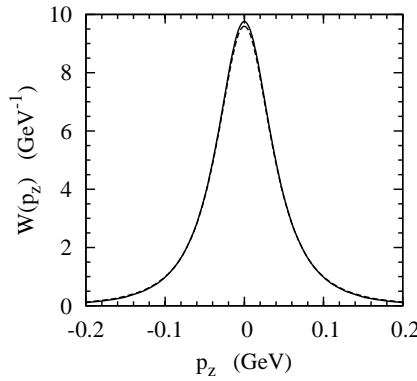


Figure 2: Distribution of the longitudinal momentum in the deuteron. Solid and dashed lines: Hulthén and CD-Bonn wave functions.

The Hulthén distribution for  $W(p_z)$  gives the following average longitudinal momentum of nucleons in the deuteron:

$$\langle p_z^2 \rangle^{1/2} = 53.9 \text{ MeV}/c \quad (10)$$

(it is  $54.9 \text{ MeV}/c$  for the CD-Bonn wave function). It also gives the following spread for the effective photon energy seen by the moving nucleon:

$$\Delta\omega^{\text{eff}} = \omega \frac{\langle p_z^2 \rangle^{1/2}}{M} = 0.057\omega. \quad (11)$$

In other words, this value characterizes the “energy resolution of the deuteron” as a “spectral measuring device” for the neutron. For  $\omega \sim 1 \text{ GeV}$  only an average of the nucleon cross section over the range  $\sim \pm 60 \text{ MeV}$  can be inferred from the deuteron data. Determination of  $\sigma_n(\omega)$  with the step of  $25 \text{ MeV}$  done in [6] cannot be physically justified.

## Unfolding

It is well known that the unfolding problem, i.e. solving the Fredholm integral equation (3) for the unknown “unsmeared deuteron cross section”  $\sigma(\omega) = \sigma_p(\omega) + \sigma_n(\omega)$ , cannot be solved without further assumptions on properties of the solution  $\sigma(\omega)$ . In particular, it is not possible to restore fast fluctuations in  $\sigma(\omega)$  at the energy scale  $\lesssim \Delta\omega^{\text{eff}}$ . To proceed, we make therefore a physically sound assumption that both the cross sections  $\sigma_p(\omega)$  and  $\sigma_n(\omega)$  can be approximated with a sum

of a few Breit-Wigner resonances (having fixed known standard masses and widths but unknown amplitudes, probably different for  $p$  and  $n$ ) plus a smooth background. Thus we write

$$\sigma(\omega) = \sum_i X_i f_i(\omega) \quad (12)$$

where  $f_i(\omega)$  is the basis of the expansion, i.e. either Breit-Wigner distributions or smooth functions of the total energy  $\sqrt{s}$ . We borrow specific forms of the functions  $f_i(\omega)$  from Ref. [6], Eqs. (11) and below. Then unknown coefficients  $X_i$  are determined from the fit of  $\hat{F}\sigma(\omega)$  to experimental data on  $\sigma_d(\omega)$  (at this point we assume that the correction  $\Delta\sigma_{pn}$  is already calculated).

A knowledge of  $X_i$ , with errorbars  $\delta X_i$  determined in the fit, can be directly converted to the knowledge of  $\sigma(\omega)$ , also with errorbars. In particular, writing fluctuations in the determined value of  $\sigma(\omega)$  as

$$\delta\sigma(\omega) = \sum_i \delta X_i f_i(\omega), \quad (13)$$

we have

$$\delta\sigma^2(\omega) = \sum_{ij} \delta X_i \delta X_j f_i(\omega) f_j(\omega) \quad (14)$$

and

$$\langle \delta\sigma^2(\omega) \rangle = \sum_{ij} C_{ij} f_i(\omega) f_j(\omega), \quad (15)$$

where

$$C_{ij} = \langle \delta X_i \delta X_j \rangle \quad (16)$$

is a standard covariance matrix of errors determined in the fit of  $X_i$ .

In this way the extracted unfolded cross section  $\sigma(\omega)$  can be shown as a smooth curve (corresponding to the central values of  $X_i$ ) surrounded with a band of the half-width given by Eq. (15) which represents errors in the cross section.

## Nonadditive corrections

The term  $\Delta\sigma_{pn}(\omega)$  in Eq. (3) takes into account various effects violating additivity of the photoabsorption cross sections on individual nucleons. Among them:

- interference of diagrams of photoproduction off proton and neutron, Fig. 1, leading to identical final states; the Fermi statistics of the emitted nucleons (antisymmetrization) leading to the so-called Pauli blocking,
- interaction between emitted particles (final state interaction, FSI) including both interaction of unbound nucleons and binding of nucleons (formation of the deuteron in the final state), interaction of pions (or other particles), produced on one nucleon, with the second nucleon in the deuteron,
- absorption of pions (and the presence of processes such as the deuteron photodisintegration, without pions in the final state).

Now we briefly discuss all these effects starting with the reaction of single-pion photoproduction,  $\gamma d \rightarrow \pi NN$ , considered in the model that includes diagrams of impulse approximation (Fig. 1) and the final state  $NN$  and  $\pi N$  interaction to one loop (Fig. 3). Formalism and the main building blocks of this model that was previously used in the energy region of the  $\Delta(1232)$  resonance can be found elsewhere [13, 14]. Generally, the model works well for the channel  $\gamma d \rightarrow \pi^- pp$  in the  $\Delta(1232)$  region but not so well for  $\gamma d \rightarrow \pi^0 pn$ , see Fig. 4. Reasons for the discrepancy are not clear but other authors get similar results and also cannot describe the data (see, e.g., [17]). We will not use the model for energies too close to the  $\Delta(1232)$  region.

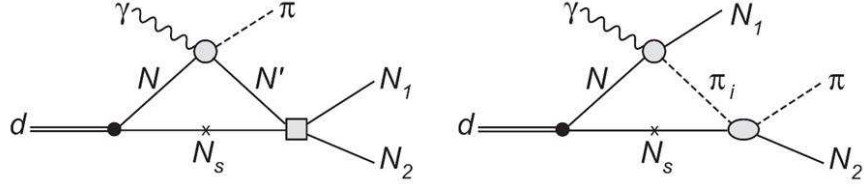


Figure 3: Diagrams with the final state  $NN$  and  $\pi N$  interaction (to one loop) for  $\gamma d \rightarrow \pi NN$ .

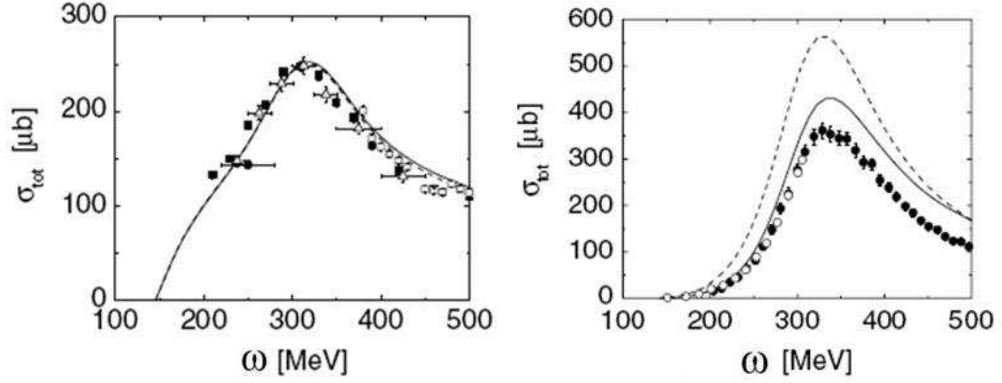


Figure 4: Model [13, 14] predictions for  $\gamma d \rightarrow \pi^- pp$  (left) and  $\gamma d \rightarrow \pi^0 pn$  (right) in the region of the  $\Delta(1232)$ .

In the present calculation that covers higher energies, “elementary” amplitudes of  $\gamma N \rightarrow \pi N$  are taken from the MAID analysis [15] (with a proper off-shell extrapolation); those for  $NN \rightarrow NN$  are taken from the analysis of SAID [16] (again with an off-shell extrapolation). In the following plots we show obtained results for  $\Delta\sigma_{pn}(\omega)$  in different isotopic channels.

### 1. Interference contributions from diagrams of impulse approximation for $\gamma d \rightarrow \pi NN$

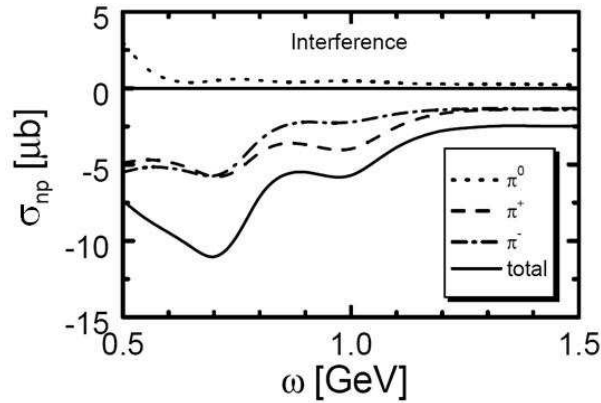


Figure 5: Contribution to  $\Delta\sigma_{pn}$  due to interference of diagrams in Fig. 1 of impulse approximation for  $\gamma d \rightarrow \pi NN$ .

## 2. $NN$ FSI interaction in $\gamma d \rightarrow \pi NN$ and $\gamma d \rightarrow \pi d$

We put here  $NN$  FSI contributions for the continuous and bound states together because there is a tendency for their cancelation that can be traced to the unitarity (closure). The matter is that the  $NN$  interaction in the continuous spectrum can be thought as a replacement of the plane  $NN$  wave in the reaction amplitude of the plane-wave impulse approximation,

$$T^{\text{PWIA}}(E_{NN}) = \langle NN | T(\gamma N \rightarrow \pi N) | d \rangle, \quad (17)$$

with the distorted  $NN$  wave in the reaction amplitude of the distorted-wave impulse approximation,

$$T^{\text{DWIA}}(E_{NN}) = \langle \psi^{(-)}(NN) | T(\gamma N \rightarrow \pi N) | d \rangle. \quad (18)$$

Here we explicitly indicate the energy of the  $NN$  state. Also, the coherent amplitude, with the final bound  $NN$  system, is

$$T^{\text{coh}}(E_d) = \langle d | T(\gamma N \rightarrow \pi N) | d \rangle. \quad (19)$$

Owing to the closure, i.e. a completeness of eigen states of the free  $NN$  Hamiltonian as well as those of a Hamiltonian with  $NN$  interaction,

$$1 = \sum_{NN, E_{NN}} |NN\rangle \langle NN| = \sum_{NN, E_{NN}} |\psi^{(-)}(NN)\rangle \langle \psi^{(-)}(NN)| + \sum_d |d\rangle \langle d|, \quad (20)$$

the square of the PWIA off-shell amplitude integrated over all possible  $NN$  states, irrespectively to their energies, exactly coincides with the square of the DWIA off-shell amplitude (also integrated over all possible states) plus the square of the coherent amplitude. In case when a subset of  $NN$  states of certain energies is only considered, as in the case of finding cross sections at a certain energy, the coincidence of  $|T^{\text{PWIA}}|^2$  with  $|T^{\text{DWIA}}|^2 + |T^{\text{coh}}|^2$  is not strictly valid, however a tendency to have a compensation between the coherent contribution to the cross section and a decrease in the DWIA cross section still remains.

An illustration of this general tendency can be found in Fig. 6 where the negative  $NN$ -FSI contribution to  $\gamma d \rightarrow \pi^0 pn$  is close in the magnitude to the positive coherent contribution to  $\gamma d \rightarrow \pi^0 d$  (see dotted curves).

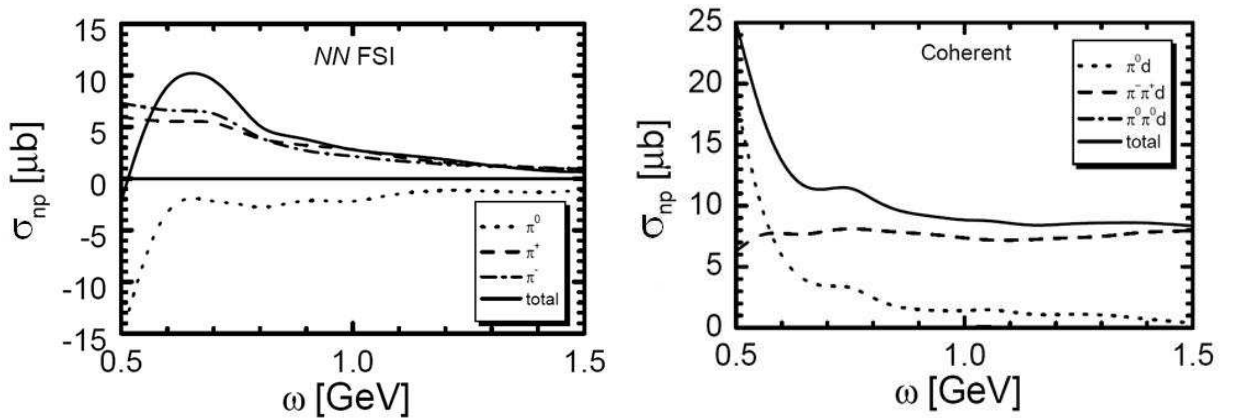


Figure 6: Left: Contribution to  $\Delta\sigma_{pn}$  due to final state  $NN$  interaction in  $\gamma d \rightarrow \pi NN$ . Right: Contribution to  $\Delta\sigma_{pn}$  from  $\gamma d \rightarrow \pi^0 d$  and  $\gamma d \rightarrow \pi\pi d$ .

### 3. $NN$ FSI interaction in $\gamma d \rightarrow \pi\pi NN$ and $\gamma d \rightarrow \pi\pi d$

Consideration of the reactions  $\gamma d \rightarrow \pi\pi NN$  and  $\gamma d \rightarrow \pi\pi d$  is similar but more involved owing to a more complicated structure of the elementary  $\gamma N \rightarrow \pi\pi N$  amplitude. We rely here on results obtained by Fix and Arenhövel [19,20] from which we infer contributions to  $\Delta\sigma_{pn}$  shown in Figs. 6 (the right panel) and 7. Again we see an essential partial cancelation between  $\gamma d \rightarrow \pi^+\pi^-d$  and  $NN$ -FSI effects in  $\gamma d \rightarrow \pi^+\pi^-pn$ .

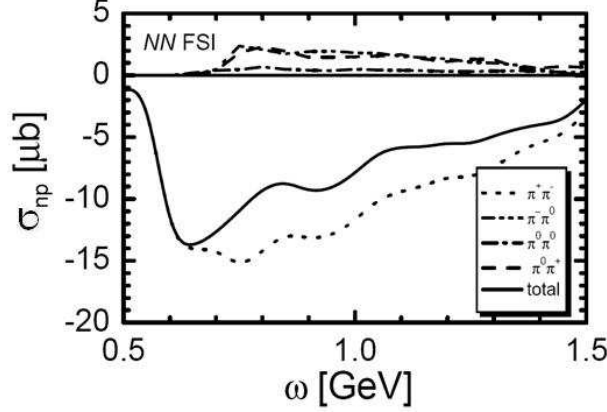


Figure 7: Final state  $NN$  interaction in  $\gamma d \rightarrow \pi\pi NN$ .

### 4. Other small contributions and the net result for $\Delta\sigma_{pn}$

We do not show contributions to  $\Delta\sigma_{pn}$  from  $\pi N$  FSI in  $\gamma d \rightarrow \pi NN$  (found in the described model) and contributions from the deuteron photodisintegration,  $\gamma d \rightarrow pn$  (it can be directly found from experimental data of CLAS [18]) because they are rather small with the exception for energies close to the  $\Delta(1232)$  resonance region. We can anticipate that  $\Delta\sigma_{pn}$  is not affected by  $\eta$  meson photoproduction because  $\eta N$  interaction is weaker than that of  $\pi N$  and because effects of  $NN$  FSI interaction in the continuum and in the bound state are again nearly canceled.

Taking all contributions together, we arrive at the total value of  $\Delta\sigma_{pn}$  shown in Fig. 8 which is the main result of this section. In spite of quite a few pieces of order  $10 \mu\text{b}$ , the sum of all contributions to  $\Delta\sigma_{pn}$  is found surprisingly small, so that our improvement to the unfolding procedure is mainly reduced to a refinement in solving the integral equation.

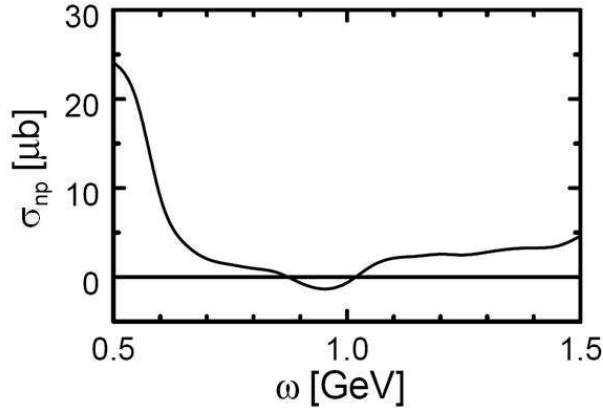


Figure 8: Total value of  $\Delta\sigma_{pn}$ .

## Extraction of the photoabsorption cross section on the neutron

Known now all ingredients of Eq. (3), we can fit experimental data, determine the unsmeared deuteron cross section  $\sigma_p + \sigma_n$  and then find the neutron cross section  $\sigma_n$ . We illustrate this procedure using Daresbury data [5, 6] for the proton and the deuteron.

Figure 9 (the left panel) shows a smooth fit (the curve labeled “tot”) with Eq. (12) to the experimental proton data and the result of its smearing with the smearing operator  $\hat{F}$ . Separately shown is the contribution of resonances (and its smearing) and a smooth background. At the right panel of Fig. 9 a fitting curve is shown that, after smearing and adding  $\Delta\sigma_{pn}$ , comes through experimental data points (the curve labeled “totF”).

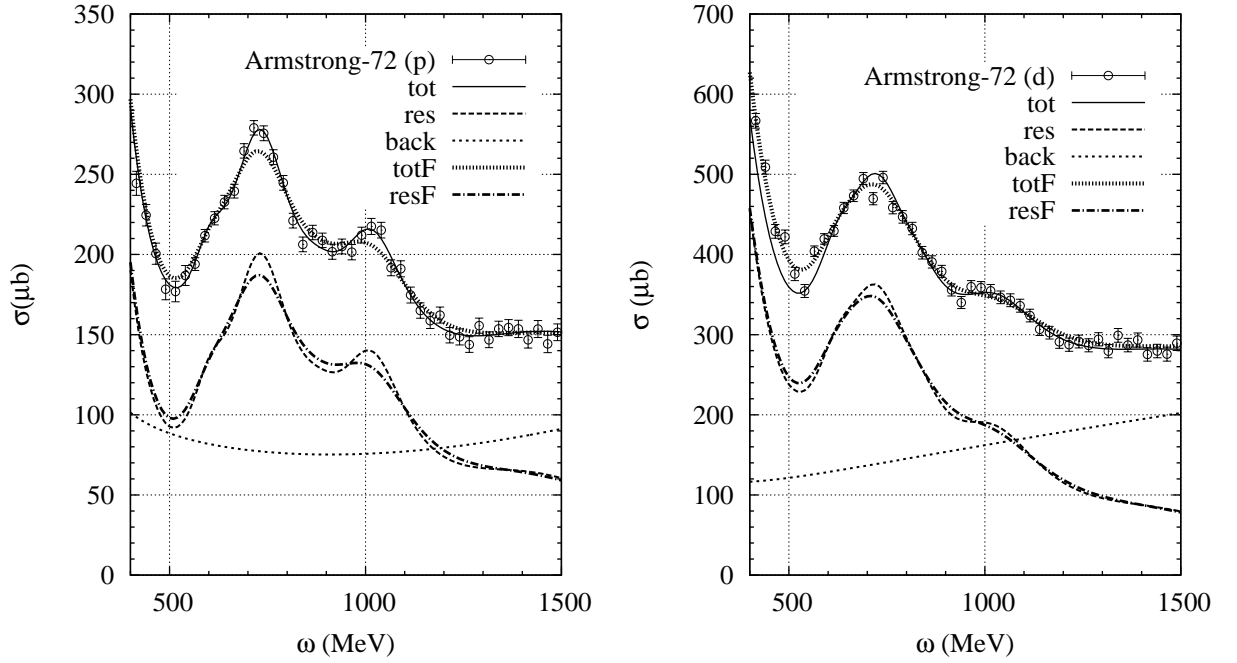


Figure 9: Daresbury data for the proton (left) [5] and the deuteron (right) [6], their fit and smearing.

From this fit the neutron cross section can be found as a difference, see Fig. 10 (the left panel). In a similar way the neutron cross section can be found from Mainz data [21]. Our results are shown in Fig. 10 (the right panel). Bands indicate errors in the found neutron cross sections there.

## Conclusions

An improved procedure of extracting the total photoabsorption cross section on the neutron from data on the deuteron is proposed. It involves a more correct treatment of folding/unfolding of the Fermi smearing of individual nucleon contributions.

Non-additive corrections are evaluated at medium energies where VMD does not yet work. They are relatively small in total but they might be more important in analyses of partial channels of photoabsorption.

We hope that the obtained results will be useful for interpretation of the GRAAL data and future experiments.

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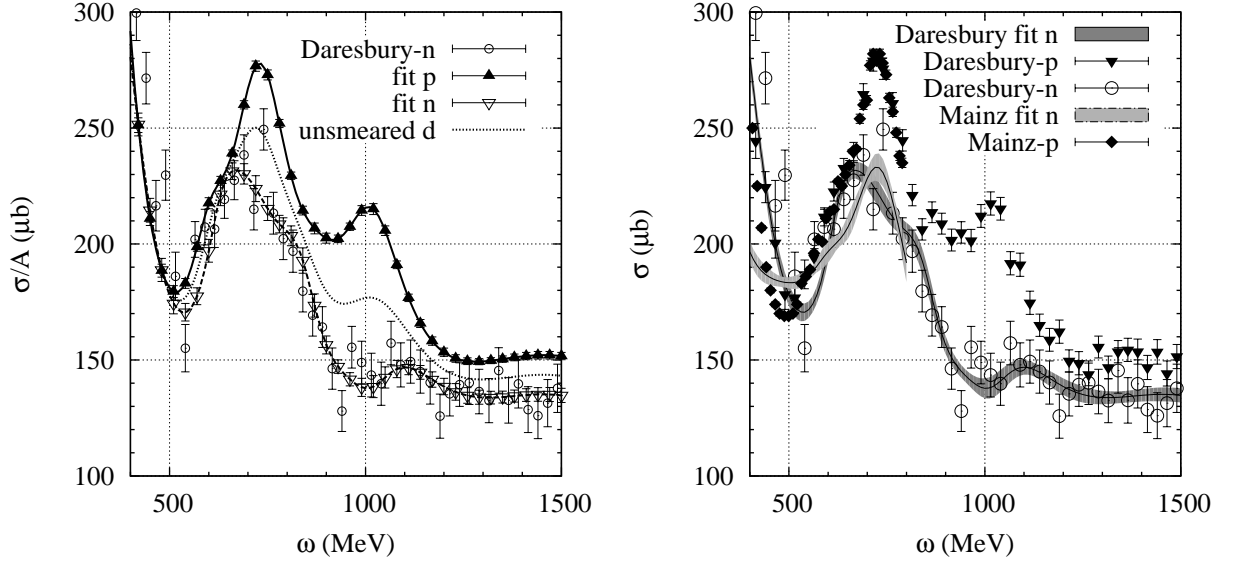


Figure 10: Extraction of the neutron cross section  $\sigma_n$  from the deuteron data (Daresbury [6] and Mainz [21]). Original values of  $\sigma_n$  from the Daresbury experiment are shown with open circles.

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